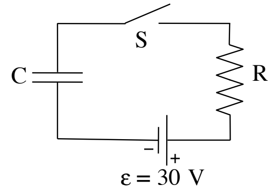
Problem 28.34



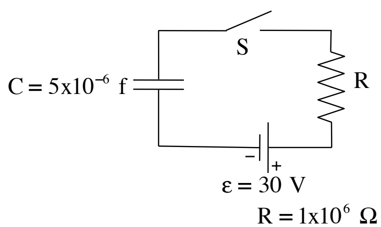
The circuit's parameters are $R = 1\text{x}10^{6}\ \Omega$, $C = 5\text{x}10^{-6}\ \text{f}$ and $\varepsilon = 30\ \text{V}$.

a.) Time constant:

$$\begin{aligned}\tau &= RC \\ &= \left(1\text{x}10^{6}\ \Omega\right)\left(5\text{x}10^{-6}\ \text{f}\right) \\ &= 5\ \text{seconds}\end{aligned}$$

b.) Once the capacitor is completely charged, no current will flow through the circuit and all of the battery's voltage drop will be across the capacitor. In that case, we can write:

$$\begin{aligned}Q_{max} &= C\varepsilon \\ &= \left(5\text{x}10^{-6}\ \text{f}\right)\left(30\ \text{V}\right) \\ &= 150\text{x}10^{-6}\ \text{coulombs}\end{aligned}$$

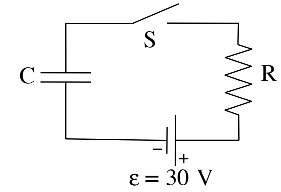


c.) There are two ways to do this. The first is to use the current function we've derived:

$$\begin{aligned}i(t) &= i_o e^{-t/RC} \\ &= \frac{\varepsilon}{R} e^{-t/RC} \\ &= \frac{\left(30\ \text{V}\right)}{\left(1\text{x}10^{6}\ \Omega\right)} e^{-t/\left(1\text{x}10^{6}\ \Omega\right)\left(5\text{x}10^{-6}\ \text{f}\right)} \\ &= \left(30\text{x}10^{-6}\ \text{A}\right)e^{-t/5}\end{aligned}$$

Evaluating this for t = 10 seconds yields:

$$\begin{aligned}i &= \left(30\text{x}10^{-6}\ \text{A}\right)e^{-10/5} \\ &= 4.06\text{x}10^{-6}\ \text{A}\end{aligned}$$



The second way: Two time constants is the amount of time it takes for the current to drop to approximately 13.5% of its initial value. In Part "a," we determine that the time constant was 5 seconds. That means 10 seconds is two time constants worth, which means our current value at that point should be:

$$\begin{aligned}i(2\tau) &= \left(.135\right)i_o \\ &= \left(.135\right)\frac{\varepsilon}{R} \\ &= \left(.135\right)\frac{\left(30\ \text{V}\right)}{\left(1\text{x}10^{6}\ \Omega\right)} \\ &= 4.05\text{x}10^{-6}\ \text{A}\end{aligned}$$

The slight deviation between the two values is due to round-off error.